Constants

a Double Choco puzzle pack

by Lavaloid

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Chapter 1 | Introduction

1.1 Remarks

This pack started as a section in my 2K Double Choco (part of archive #2000 in the Cracking the Cryptic Discord server), where I had the given numbers resemble the digits of π. On 15th May, 2021, I set the e puzzle that is part of this pack. On 30th June, I set the π puzzle, which is what led to the creation of this pack. Thanks to everyone who tested the puzzles and proofread this document, and have fun solving!

P.S. Most of the facts are shamelessly stolen from Wikipedia and should be taken with a grain of salt. Do fact-check these and let me know if anything is wrong or misrepresented! You can contact me on Discord (`Lavaloid#4388`) or Twitter (`@LavaloidPuzzles`).

1.2 Document Structure

Chapter 1 (this chapter) contains my comments on this puzzle pack, the rules, and examples of correct and wrong solutions. Chapter 2 contains each puzzle, some facts about the constant used, and a link to a `puzz.link` player. On the very last page is the one-page version of this pack.

1.3 Rules

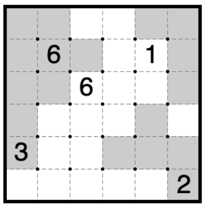
This description is written by Eric Fox with small modifications from me.

1. Divide the grid into regions of orthogonally connected cells, each containing some connected white cells and some connected grey cells, such that the shape of the white cells is identical to the shape of the grey cells, allowing rotations and reflections.

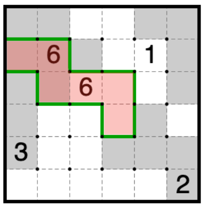
2. Clued cells must belong to a region containing the indicated number of white cells and the indicated number of grey cells. A region may contain zero, one, or multiple numbers.

3. **The number zero (0) is only for aesthetics and should be ignored.**

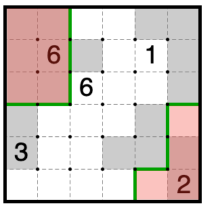
1.4 Examples



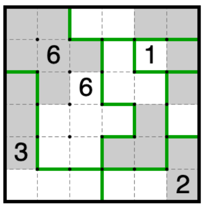
An example of a Double Choco.



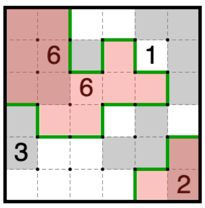
Region size (3) does not match given clue (6)



Top left: Has no white regions.
Bottom right: Has two white regions.



A finished Double Choco.



Top left: Shape does not match.
Bottom right: Size does not match.

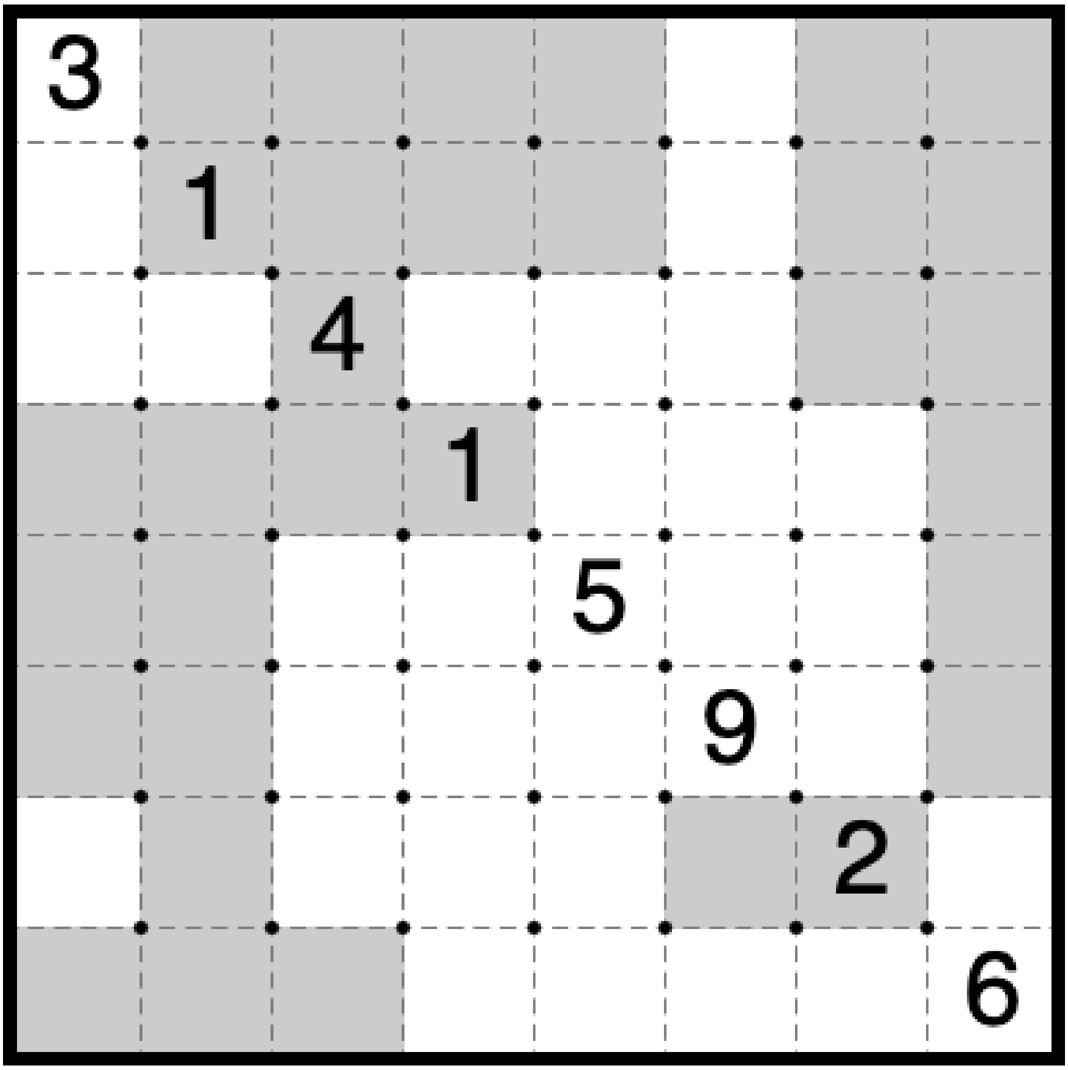
Chapter 2 | Puzzles

2.1 Pi (π)

Most people who are solving this pack already knows what π is, but just in case: π is a constant describing the ratio between the circumference of a circle and its diameter. Its approximate value is

$$\pi = 3.1415926...$$

The history of π goes back thousands of years. A clay tablet found in Babylon that estimates $\pi \approx \frac{25}{8} = 3.125$ is dated between 1900 BC and 1600 BC.



Link: `https://puzz.link/p?dbchoco/8/8/fdti7se1o53e03n1n4n1n5n9n2n6`

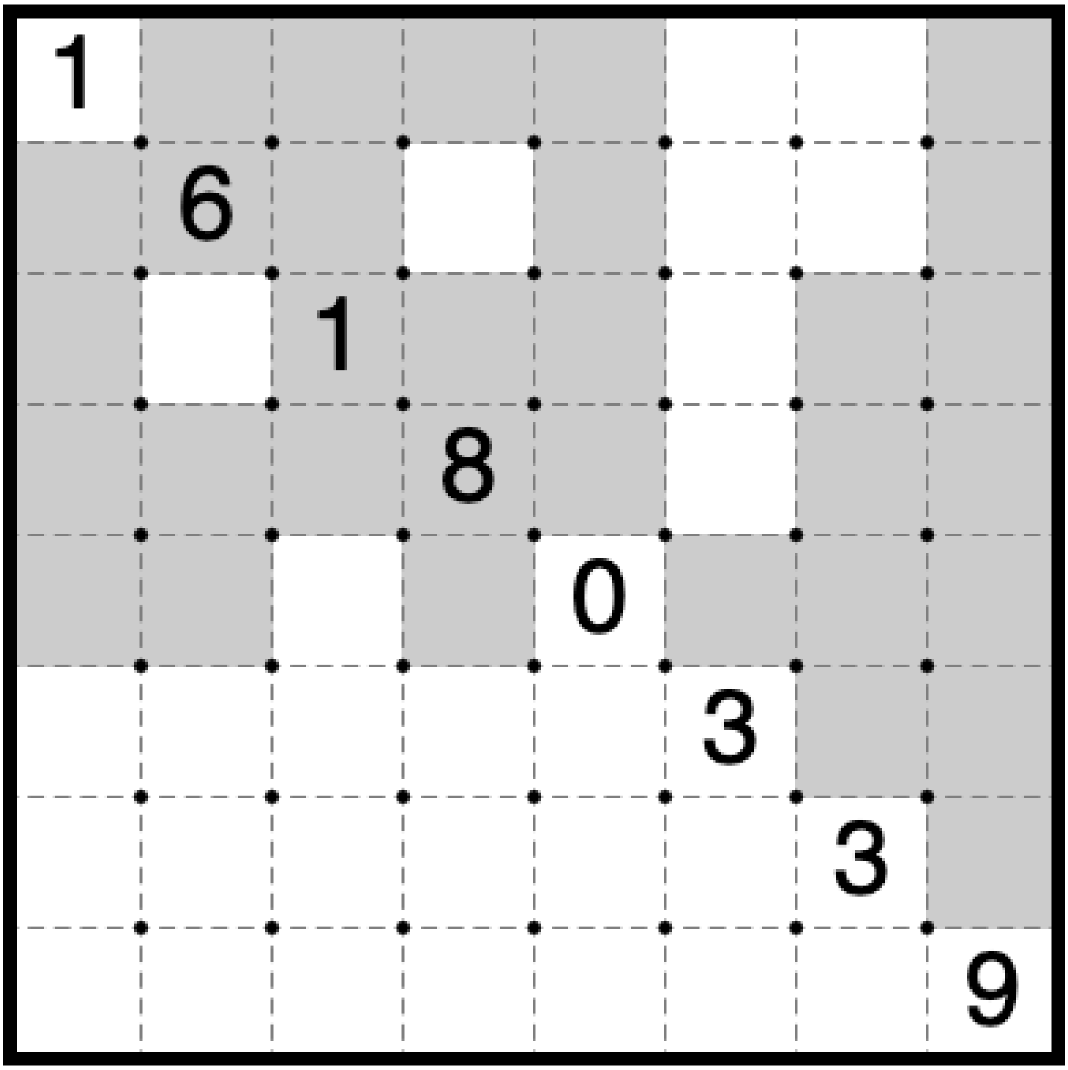
2.2 Golden ratio (φ)

The golden ratio is another constant that has been known for a very long time. The first known definition of φ is found in Euclid's *Elements* (300 BC):

> *A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser.*

In other words, φ is the value of $\frac{a}{b}$ where a, b satisfy $\frac{a+b}{a} = \frac{a}{b}$ and $a > b$. The value of the golden ratio is

$$\varphi = \frac{1+\sqrt{5}}{2} = 1.6180339...$$



Link: `https://puzz.link/p?dbchoco/8/8/f7krnuun0c0g01n6n1n8n0n3n3n9`

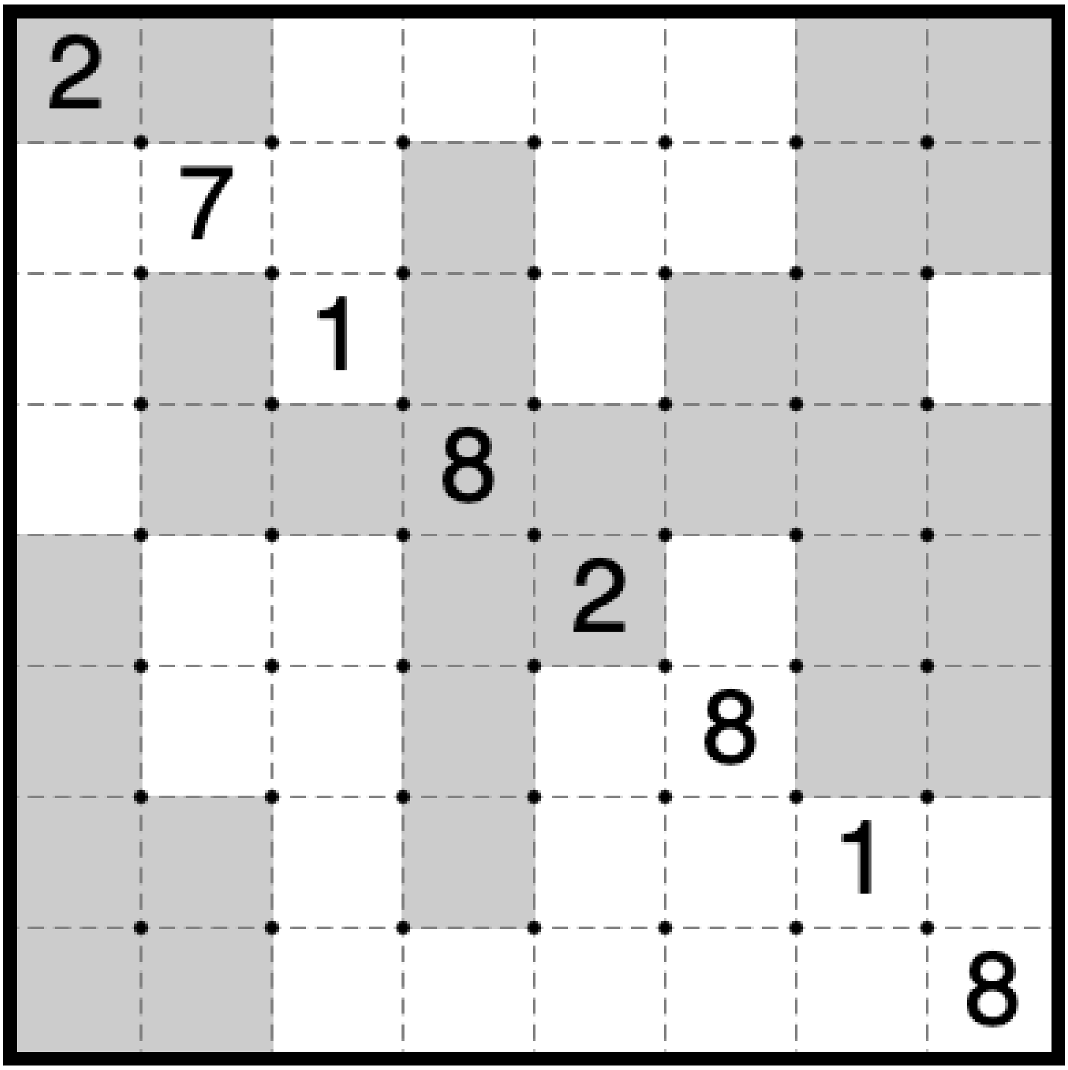
2.3 Euler's number (e)

Euler's number is one of the most important constants in calculus. As the name suggests, the discovery of the constant is often credited to Jacob Bernoulli in 1683. It was shown in the context of the compound interest problem:

> *An account starts with $1.00 and pays 100% interest per year. If the interest is credited once, at the end of the year, the value of the account at year-end will be $2.00. What happens if the interest is computed and credited more frequently during the year?*

Bernoulli noticed that as the frequency becomes higher, the value approaches a limit. There are many definitions for e, but in this context it is

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = 2.7182818...$$

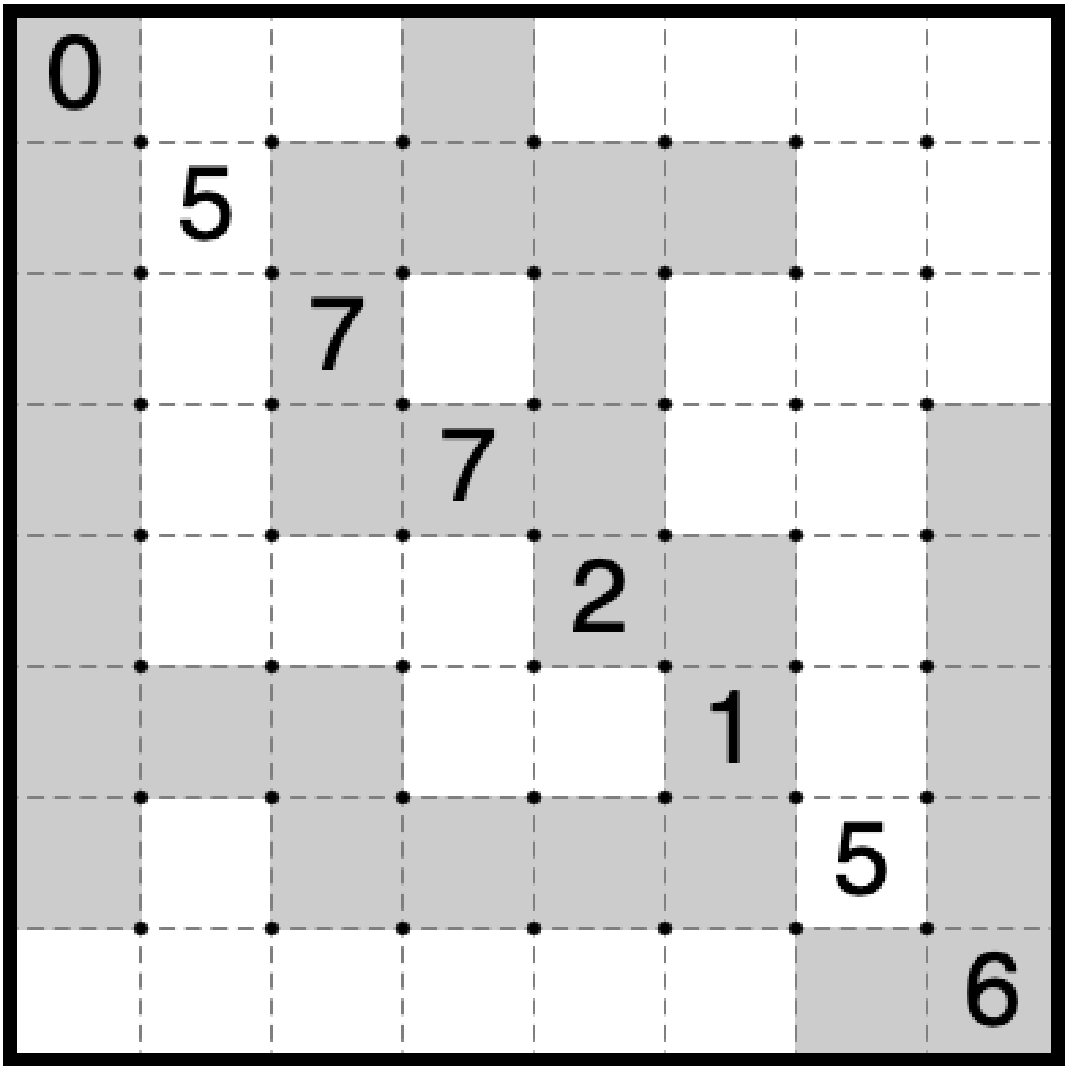


Link: `https://puzz.link/p?dbchoco/8/8/oc9lcvsrif8c02n7n1n8n2n8n1n8`

2.4 Euler-Mascheroni constant (γ)

The Euler-Mascheroni constant was discovered by Euler in 1734. Lorenzo Mascheroni's name was attached after his work in 1790. The constant is defined as the limiting difference between the partial sums of the harmonic series and the natural logarithm, in other words:

$$\gamma = \lim_{n\to\infty} \left(\sum_{k=1}^{n} \frac{1}{k} \right) - \ln n = \int_0^{\infty} \left(\frac{1}{\lfloor x \rfloor} - \frac{1}{x} \right) dx = 0.5772156...$$



Link: `https://puzz.link/p?dbchoco/8/8/i2uahecdsmug60n5n7n7n2n1n5n6`

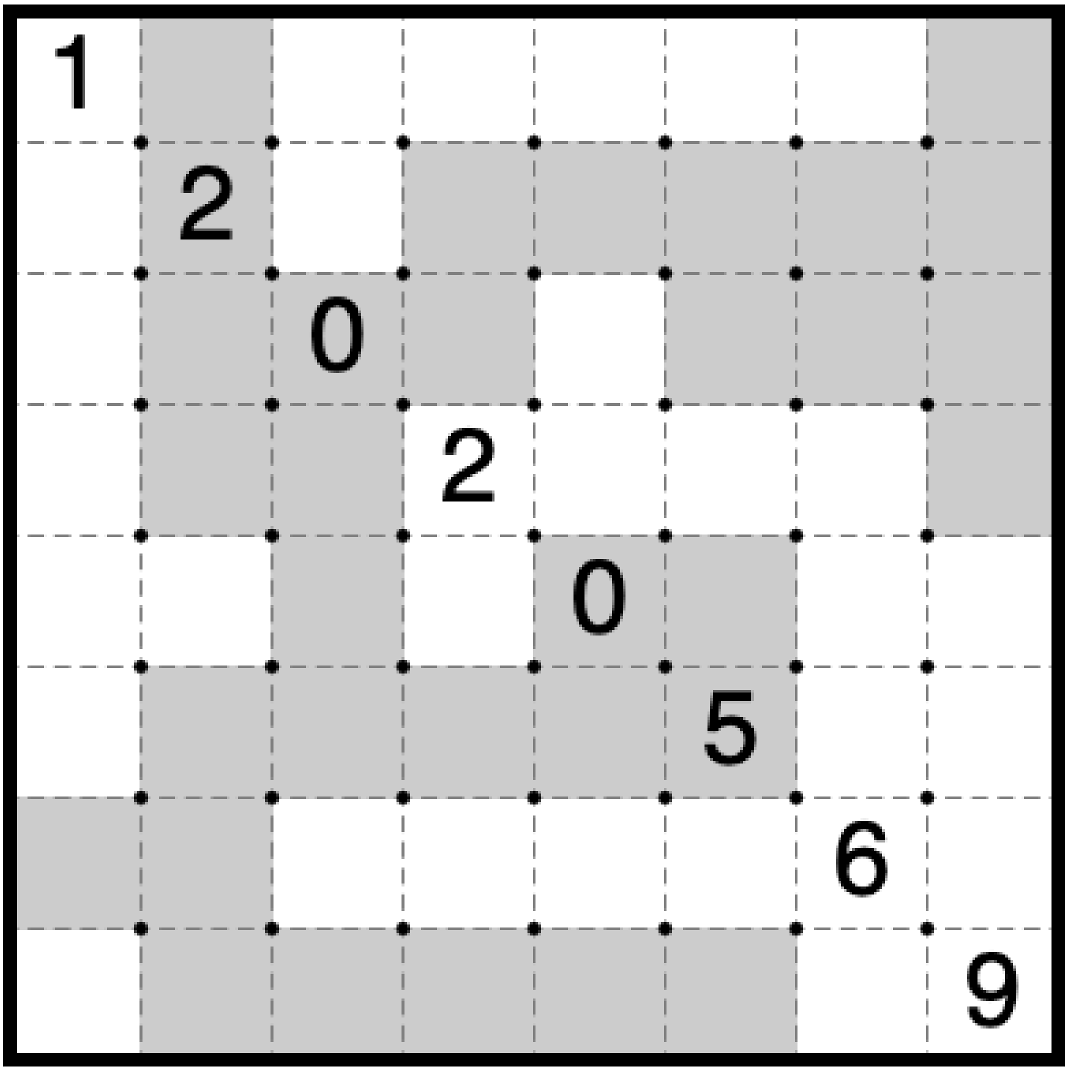
2.5 Apéry's constant ($\zeta(3)$)

Apéry's constant is named after Roger Apéry, who proved its irrationality in 1978. The function $\zeta(s)$ refers to Riemann's zeta function, which is defined as

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

for all complex numbers s with the real part greater than 1. The value of Apéry's constant is

$$\zeta(3) = \sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \cdots = 1.2020569...$$



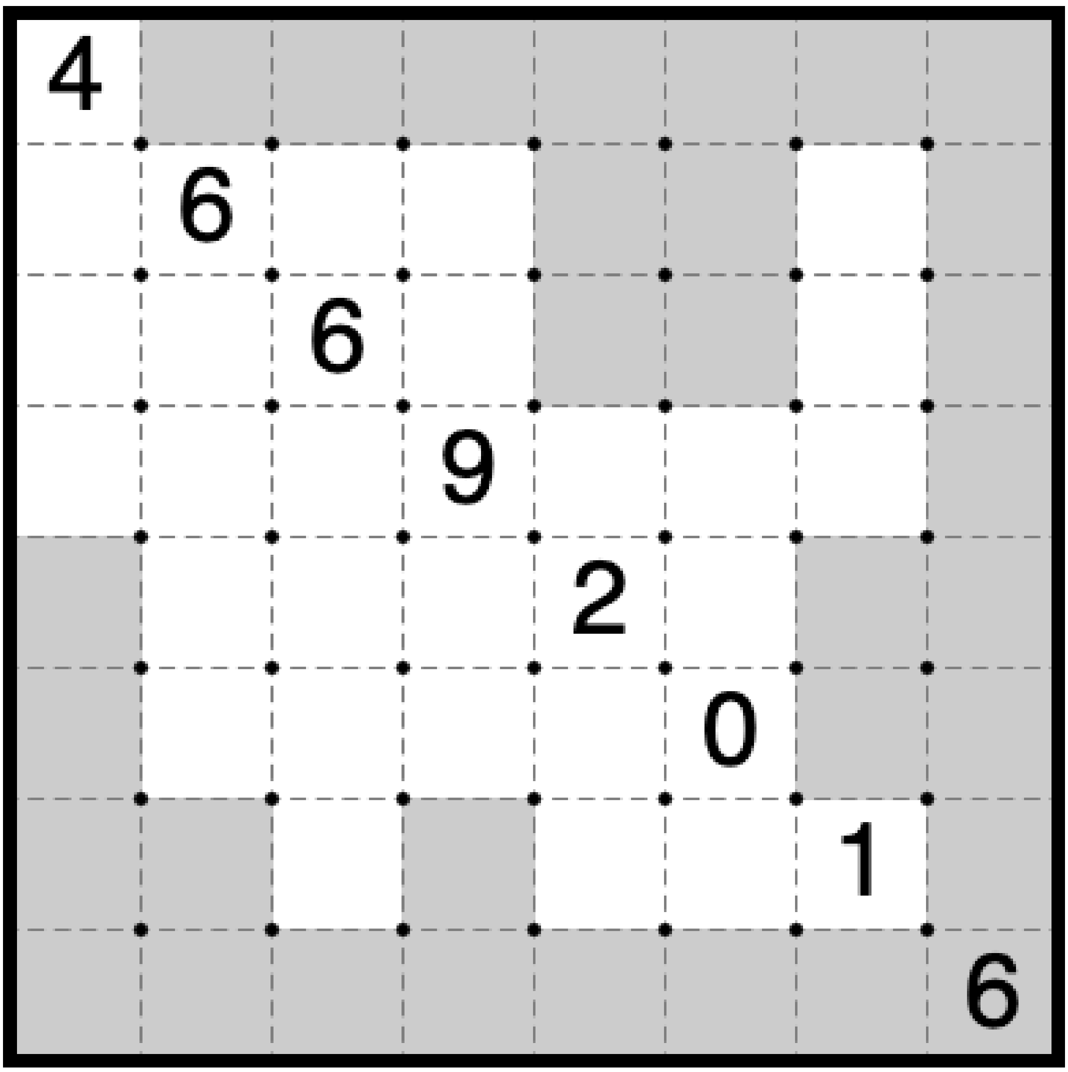
Link: `https://puzz.link/p?dbchoco/8/8/85fneo9cfj07o1n2n0n2n0n5n6n9`

2.6 Feigenbaum's first constant (δ)

To define Feigenbaum's first constant, we should first define a period-doubling bifurcation. Consider the series $x_{n+1} = rx_n(1 - x_n)$ which has a starting condition x_0 and growth rate r. From $r = 3$ onwards, the series will oscillate with a certain period; 2, 4, 8, and so on; doubling at various points before becoming chaotic on $r \approx 3.56695$. The moment the period doubles is called a *period-doubling bifurcation*[1].

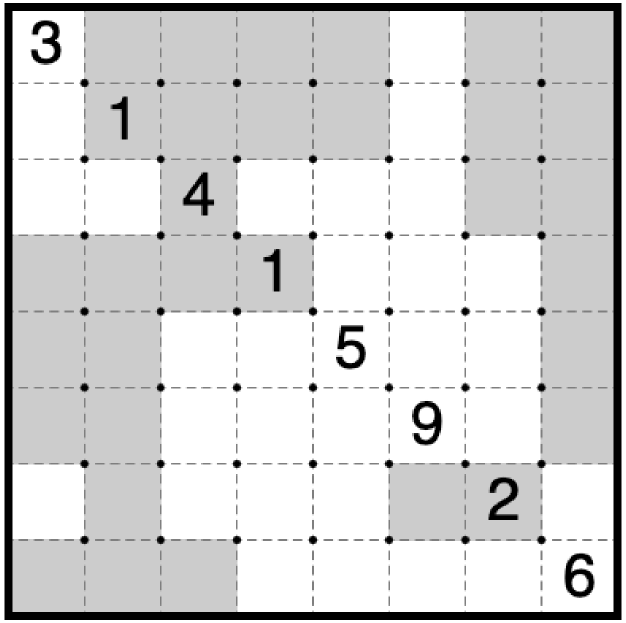
In 1975, Mitchell J. Feigenbaum discovered the following: let's label the values r at which this happens a_1, a_2, a_3, and so on and consider the distance between consecutive values a_n. In fact, the ratio between two consecutive distances converge to a single value! This value is what we call Feigenbaum's first constant:

$$\delta = \lim_{n\to\infty} \frac{a_{n-1} - a_{n-2}}{a_n - a_{n-1}} = 4.6692016...$$

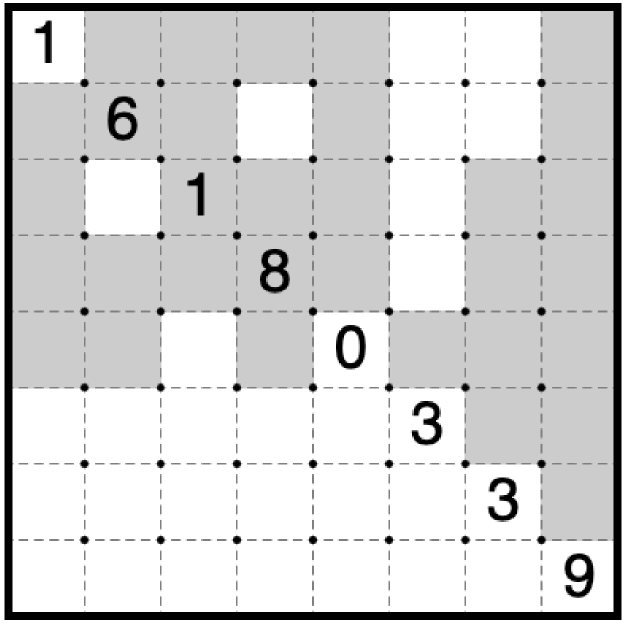


Link: `https://puzz.link/p?dbchoco/8/8/fs6gq0c3gf8vu4n6n6n9n2n0n1n6`

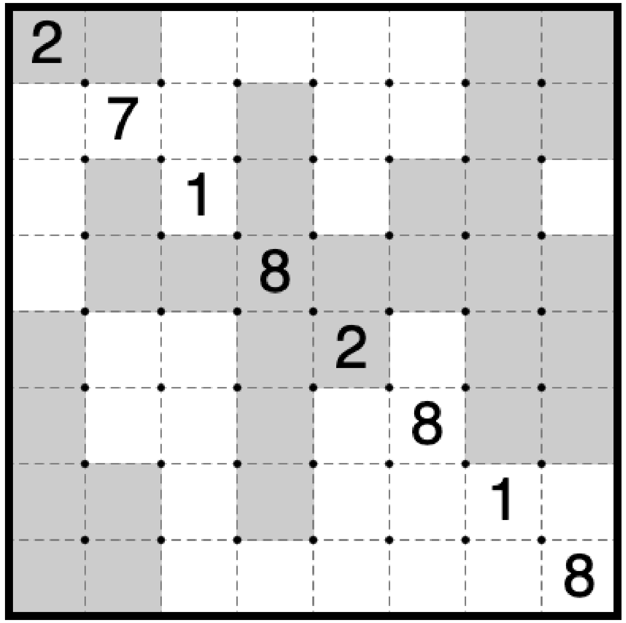
[1]I have a marvellous puzzle-themed pun related to this, which this footnote is too narrow to contain.



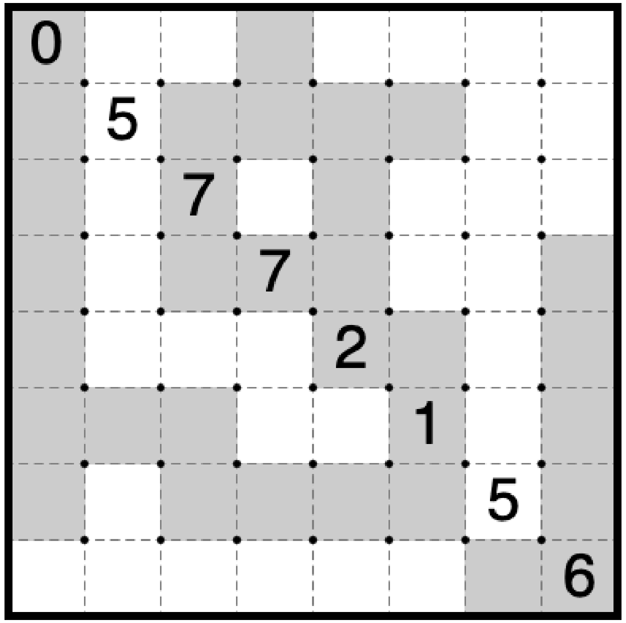
(1) Pi (π)



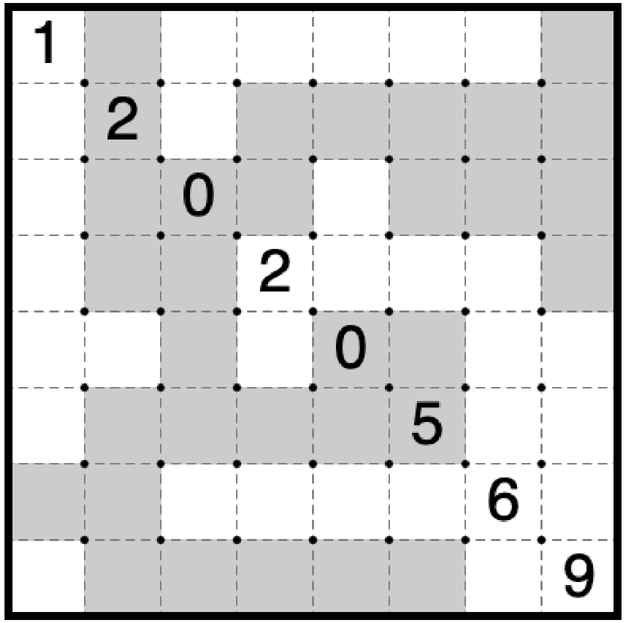
(2) Golden ratio (φ)



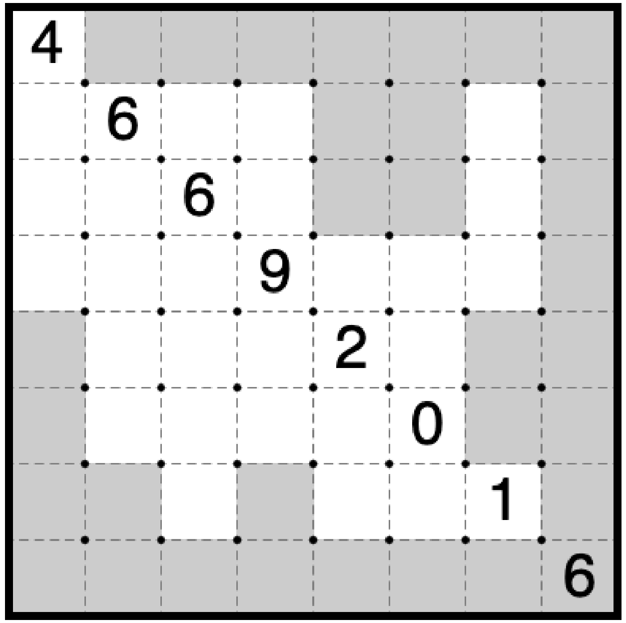
(3) Euler's number (e)



(4) Euler-Mascheroni constant (γ)



(5) Apéry's constant ($\zeta(3)$)



(6) Feigenbaum's first constant (δ)